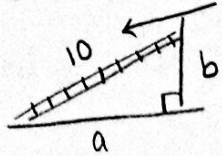
8. The base of a 10-ft ladder that is leaning against a wall is pushed towards the wall. When the base is 6 ft from the wall and moving at the rate of 2 ft/sec, how fast is the top of the ladder sliding up the wall?

not changing!

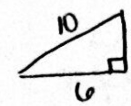
1. 

2. want $\frac{db}{dt}$ when $a=6$, $\frac{da}{dt}=-2$.

3. $a^2+b^2=10^2$

4. $\cancel{2}a\frac{da}{dt}+\cancel{2}b\frac{db}{dt}=0$

5. $6(-2)+8\frac{db}{dt}=0$

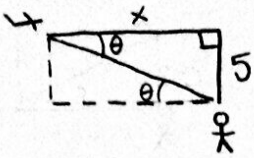
When $a=6$: 

$b=\sqrt{10^2-6^2}=8$

6. $\frac{db}{dt}=\frac{12}{8}=\frac{3}{2}$

7. The top of the ladder is sliding up the wall at a rate of $\boxed{\frac{3}{2}}$ ft/sec.

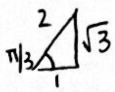
9. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation is changing when the angle is:

(a) $\frac{\pi}{3}$ ① 

② Want $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{3}$, $\frac{dx}{dt} = -600$

③ $\cot\theta = \frac{x}{5}$ ⟵ use $\cot\theta$ so x is in the numerator

④ $-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

⑤ $-\csc^2\left(\frac{\pi}{3}\right)\frac{d\theta}{dt} = \frac{1}{5}(-600)$ 

$\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

⑥ $-\left(\frac{2}{\sqrt{3}}\right)^2\frac{d\theta}{dt} = -120$

$$\frac{d\theta}{dt} = \frac{-120}{-4/3} = 90$$

⑦ $\boxed{90}$ rad/hour

(b) $\frac{\pi}{4}$ ① same

② Want $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{4}$, $\frac{dx}{dt} = -600$

③ same

④ same

⑤ $-\csc^2\left(\frac{\pi}{4}\right)\frac{d\theta}{dt} = \frac{1}{5}(-600)$ 

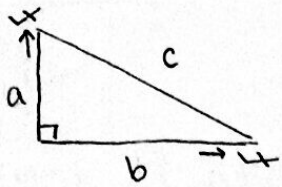
$\csc\frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$

⑥ $-(\sqrt{2})^2\frac{d\theta}{dt} = -120$

$$\frac{d\theta}{dt} = \frac{-120}{-2} = 60$$

⑦ $\boxed{60}$ rad/hr

10. Two airplanes depart the Purdue Airport. One leaves at noon heading due east at 550 miles per hour and the other leaves at 12:30pm heading due north at 600 miles per hour. How quickly is the distance between them changing at 1:30pm?

① 

② Want $\frac{dc}{dt}$ when $b = 1.5(550) = 825$, $a = 1(600)$, $\frac{db}{dt} = 550$, $\frac{da}{dt} = 600$

③ $a^2 + b^2 = c^2$

④ $\cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt} = \cancel{2}c\frac{dc}{dt}$

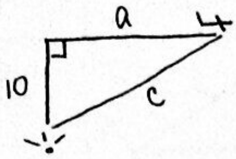
⑤ $600(600) + 825(550) = 25\sqrt{1665}\,\frac{dc}{dt}$ $\longrightarrow$ $c = \sqrt{825^2 + 600^2} = 25\sqrt{1665}$

⑥ $\frac{dc}{dt} = \frac{825(550) + 600(600)}{25\sqrt{1665}}$

$= \frac{32{,}500}{\sqrt{1665}} \cdot \frac{\sqrt{1665}}{\sqrt{1665}} = \frac{32{,}500\sqrt{1665}}{1665} = \frac{6500\sqrt{1665}}{333}$

⑦ $\boxed{\frac{32{,}500}{\sqrt{1665}}}$ miles/hr

11. An airplane flying at an altitude of 10 miles passes directly over a radar antenna. When the airplane is 15 miles away, the radar detects that the distance is changing at a rate of 250 miles per hour. What is the speed of the airplane?

1. 

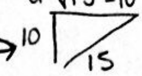
2. want $\frac{da}{dt}$ when $c = 15$, $\frac{dc}{dt} = 250$

3. $a^2 + 10^2 = c^2$

4. $\cancel{2}a \frac{da}{dt} = \cancel{2}c \frac{dc}{dt}$

5. $5\sqrt{5} \frac{da}{dt} = 15(250)$

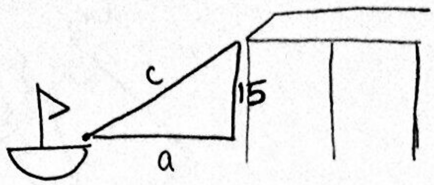
$a = \sqrt{15^2 - 10^2} = \sqrt{125} = 5\sqrt{5}$



6. $\frac{da}{dt} = \frac{750}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{750\sqrt{5}}{5} = 250\sqrt{5}$

7. $\boxed{250\sqrt{5}}$ miles/hour

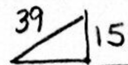
12. A boat is pulled into a dock by means of a winch 15 feet above the deck of the boat. The winch pulls in rope at a rate of 5 feet per second. Determine the speed of the boat when there is 39 feet of rope out.

① 

② Want $\frac{da}{dt}$ when $c = 39$, $\frac{dc}{dt} = -5$

③ $a^2 + 15^2 = c^2$

④ $\cancel{2}a\frac{da}{dt} = \cancel{2}c\frac{dc}{dt}$

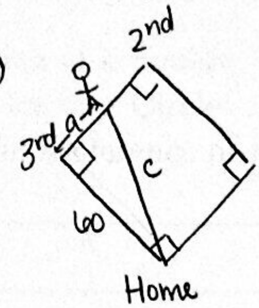
⑤ $36\frac{da}{dt} = 39(-5)$ → 

$a = \sqrt{39^2 - 15^2} = 36$

⑥ $\frac{da}{dt} = \frac{\overset{13}{\cancel{39}}(-5)}{\underset{12}{\cancel{36}}} = \frac{-65}{12}$

⑦ $\boxed{\frac{65}{12}}$ ft/sec (since speed is positive)

13. In softball, the distance between each base is 60 feet. A player is running from second base to third base at a speed of 16 feet per second. Find the rate at which the distance from home plate is changing when the player is 20 feet from second base.

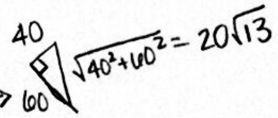
① 

② Want $\frac{dc}{dt}$ when $a = 60 - 20 = 40$, $\frac{da}{dt} = -16$

③ $a^2 + 60^2 = c^2$

④ $\cancel{2}a \frac{da}{dt} = \cancel{2}c \frac{dc}{dt}$

⑤ $40(-16) = 20\sqrt{13} \frac{dc}{dt}$



⑥ $\frac{dc}{dt} = \frac{-\cancel{40}^{2}(16)}{\cancel{20}\sqrt{13}} = \frac{-32}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{-32\sqrt{13}}{13}$

⑦ $\boxed{-\frac{32\sqrt{13}}{13}}$ ft/sec